Solution of Dynamic Programming Problem Based on LINGO—Take the Shortest Path Problem as an Example

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Abstract

The shortest path problem is widely used in real life. It possesses important practical value which how to solve the shortest path problem quickly. LINGO is a powerful tool for solving planning problems. This paper solves the shortest path problem based on LINGO.

Keywords: LINGO; Shortest Path Problem; Dynamic Programming Problem.

Introduction of LINGO

LINGO is the abbreviation of Linear Interactive and General Optimizer. It was introduced by Lindo System Inc. of America. It can not only solve programming problems, it contains "modeling language" and commonly used mathematical functions, but also it is the best choice for solving optimization models since it’s powerful function. LINGO plays an important role in the teaching of Management Operations Research course, but also has been widely used in scientific research and industry.

Lingo has the following features: 1) simple model representation; 2) powerful solution tools. Lingo has corresponding instructions and functions in model establishment, solution model, result display, data query, file processing and sensitivity analysis; 3) input and output selection of data are convenient; 4) powerful solution engine and tools; 5) model interaction or Create turn-key Applications; 6) extensive documentation and help features., the input is simple and the operation is convenient when solving the programming problem.

Example 1[1]: Solve the following linear programming problems:

\[
\begin{align*}
\min f(x) &= 2x_1 - 3x_2 \\
x_1 + 2x_2 &\geq 150 \\
x_1 &\geq 20 \\
2x_1 + x_2 &\leq 500 \\
x_1 &\geq 0, x_2 &\geq 0, \quad x_1 \text{ is an integer}, \quad x_2 \text{ is an integer}
\end{align*}
\]

Input program in LINGO as follows:

Model:
Min = 2*x1-3*x2;
X1+2*X2>= 150;
X1 >= 20;
2*X1+X2<=500;
@ gin (x1);
@ gin (x2);
End
Click on the button on the toolbar, we can get the results as follows:

That is, the optimal solution is $x_1 = 20, x_2 = 460$. The objective function value is $f= -1340$. This paper will introduce the application of LINGO in dynamic programming, especially in the shortest path problem.

### Introduction of the Shortest Path Problem

Dynamic programming is a method for solving optimization problem of multi-stage decision. The so-called multi-stage decision refers to such a kind of decision problem. Since its particularity, we can divide it into many stages according to time, space and other marks. Decision is needed in each stage to make the whole process optimal. Taking the shortest path problem as an example, we will illustrate the characteristics of dynamic programming and its solution method.

The shortest path problem is to find the shortest distance between two points in a given network graph. For example, site selection, equipment updating, investment, pipeline laying, some integer programming and dynamic programming problems can be reduced to the shortest path problem.

### Solving the Shortest Path Problem Based on LINGO

#### Example 2 (Shortest Route Problem)

If someone want to lay a gas pipeline from A to D, there are two intermediate stations. The first intermediate station is B1 and B2, and the second intermediate station is C1, C2 and C3. Connections between the two stations indicate that the pipeline can be laid. If there is no connection that means the pipeline cannot be laid. The digit in the middle of the connection show the length of the pipeline laid between two points. In the following figure, for example, $B_1 \xrightarrow{3} C_1$, the pipeline can be laid from B1 to C1 and the length of the pipeline is $3$. Our problem is to determine a route of the pipeline laid from A to D, so that the total length of the pipeline is the smallest (we called the problem shortest distance for short).

This problem belongs to the shortest path problem. The most common algorithms for the shortest path problem are Johnson algorithm, Floyd algorithm, Floyd-Warshall algorithm, Dijkstra algorithm, SPFA algorithm, Bellman-Ford algorithm and A* algorithm.
In this paper, LINGO is used to solve the problem. The following is the program.

model:
sets:
cities/A,B1,B2,C1,C2,C3,E/:L;
roads(cities,cities)/
A,B1 A,B2
B1,C1 B1,C2 B1,C3 B2,C1 B2,C2 B2,C3
C1,E C2,E C3,E/:D;
endsets
data:
D=3 5
2 3 1 2 3 3
4 3 4;
L=0......;
enddata
@for(cities(i)|i#GT#@INDEX(A):L(i)=@min(roads(j,i):L(j)+D(j,i)));
End

Click on the button on the toolbar to get the following results:

From the diagram above, we can see that the optimum route length from A to E is 8 (the optimum route is A→B1→C3→E after further analysis)

REFERENCES