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Well behaved Anisotropic charged fluid sphere in General Relativity

Neeraj Pant1*, N. Pradhan2
1Mathematics Department, National Defence Academy, Khadakwasla, Pune-411023, India
2Physics Department, National Defence Academy, Khadakwasla, Pune-411023, India

*Corresponding Author:
Neeraj Pant
Email: neeraj.pant@yahoo.com

Abstract: We present an anisotropic charged analogue of Heinzman’s [1] solution of the general relativistic field equations in curvature coordinates by using simple form of electric intensity E and pressure anisotropy factor \( \Delta \) that involve charge parameter \( K \) and anisotropy parameter \( \alpha \) respectively. Our solution is well behaved in all respects for all values of \( X \) lying in the range 0 < \( X \) ≤ 0.3, \( \alpha \) lying in the range 0 ≤ \( \alpha \) ≤ 5.5, \( K \) lying in the range 0 < \( K \) ≤ 8.6 and Schwarzschild compactness parameter \( u \) lying in the range 0 < \( u \) ≤ 0.379. Since our solution is well behaved for a wide ranges of the parameters, we can model many different types of ultra-cold compact stars like quark stars and neutron stars.

Keywords: General relativity ∙ Exact solution ∙ Curvature coordinates ∙ anisotropic fluid sphere ∙ Einstein-Maxwell ∙ Reissner-Nordstrom.

INTRODUCTION

Since the formulation of Einstein’s field equations researchers have been searching for the exact solutions with certain geometry satisfying all physical constraints. Such findings are important because they enable us to find the distribution of matter in the interior of stellar objects in terms of simple algebraic relations.

Even the strong electric field may also cause pressure anisotropy, Usov [2]. It is well known that the presence of some charge may avert the gravitational collapse by counter balancing the gravitational attraction by the electric repulsion in addition to the pressure gradient. Thus it is desirable to study the implications of Einstein-Maxwell field equations with reference to the general relativistic prediction of gravitational collapse. For these purposes anisotropic and charged fluid ball models are required. The external field of such ball is to be matched with Reissener-Nordstrom solution.

Dev and Gleiser [3] demonstrated that pressure anisotropy affects the physical properties, stability and structure of stellar matter. The stability of stellar bodies is improved for positive measure of anisotropy when compared to configurations of isotropic stellar objects.

Many papers have been published by several authors who obtained the parametric classes of exact solutions for perfect fluid with charge and neutral fluids. Few names are: Gupta and Maurya [4-6], Pant et al.[7], Maurya and Gupta [8], Tolman [9], Kuchowich[10-14], Pant et al. [15], etc. Moreover, the presence of charge and anisotropic pressure which is more realistic model of a super-dense star thereby, a few authors have recently done a remarkable work in curvature coordinates namely; Maharaj and Chaisi [16], Maharaj and Maartens [17], Thirukkanesh and Maharaj[18], Komathiraj and Maharaj[19], Mak M K and Harko T [20], Harko T and Mak M K [21], Singh et al [22], Maurya and Gupta[ 23], S. D. Maharaj · M. Chaisi [24].

In our solution, we choose seed solution of Heintzman[1] and found the solution by assuming appropriate functional form of charge parameter [7] as well as anisotropic parameter is such a way that the obtained solution is well behaved in all respects.

CONDITIONS FOR WELL BEHAVED SOLUTIONS

For well behaved nature of the solutions for anisotropic fluid sphere should satisfy the following conditions:

1) The solution should be free from physical and geometric singularities, i.e. it should yield finite and positive values of the central pressure, central density and nonzero positive value of \( (e^\nu)_{r=0} \) and \( (e^\lambda)_{r=0} = 1 \).
2) The solution should have positive value of ratio trace of energy stress tensor to energy density,$(P_r + 2P_r)/\rho$ and less than 1 (weak energy condition) and less than 1/3 (strong energy condition) throughout within the star, monotonically decreasing as well, Esculpi et al. [25].

3) The causality condition should be obeyed i.e. velocity of sound should be less than that of light throughout the model. In addition to the above the velocity of sound should be decreasing towards the surface i.e. $dP_r/\rho < 0$ or $d^2P_r/\rho < 0$ and $d\rho/\rho > 0$ or $d^2\rho/\rho > 0$ for $0 \leq r \leq r_b$ i.e. the velocity of sound is increasing with the increase of density and it should be decreasing outward.

4) $d\rho/\rho > 0$ should be satisfy everywhere within the ball. The adiabatic index, $\gamma = d\rho/\rho$ for realistic matter should be $\gamma \geq 1$.

5) The red shift $z$ should be positive, finite and monotonically decreasing in nature with the increase of $r$.

6) Electric field intensity $E$, such that $E_r = 0$, is taken to be monotonically increasing.

7) The anisotropy factor $\Delta$ should be zero at the center and increasing towards the surface.

**EINSTEIN-MAXWELL FIELD EQUATIONS OF ANISOTROPIC CHARGE FLUID DISTRIBUTION**

The interior metric of a static spherically symmetric matter distribution in curvature coordinates is given by,

$$ds^2 = -e^{2\lambda} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + c^2 e^{2\nu} dt^2$$

where $\lambda$ and $\nu$ are functions of $r$ only.

Einstein-Maxwell field equations of gravitation for a non empty space-time are

$$R_{ij} - \frac{1}{2} R g_{ij} = - \frac{8\pi G}{c^4} T_{ij}$$

$$= - \frac{8\pi G}{c^4} \left[ (p_\perp + \rho c^2) v^i v_j - p_r \delta^i_j + (p_r - p_\perp) \chi_{ij} + \frac{1}{4}\chi^m \left[ -F^{im}_{jm} + \frac{1}{4}\delta^i_j F_{mn} F^{mn} \right] \right]$$

(2)

Where $R_{ij}$ is Ricci tensor, $T_{ij}$ is energy-momentum tensor, $R$ the scalar curvature, $E_{ij}$ is the electromagnetic field tensor, $p_r$ and $p_\perp$ denotes radial and transverse pressure, $\rho$ the density distribution, $v_i$ the four velocity and $\chi_{ij}$ is the unit space-like vector in radial direction.

For the metric eq. (1) the Einstein-Maxwell’s field equations (2) of gravitation for a nonempty space-time reduces to the following set of relevant equations

$$\frac{8\pi G}{c^4} p_r = \frac{\nu'}{r} e^{-\lambda} - \frac{1-e^{-\lambda}}{2r^2} + \frac{q^2}{r^4}$$

(3)

$$\frac{8\pi G}{c^4} p_\perp = e^{-\lambda} \left( \frac{v^i v_j}{2} - \frac{\lambda'}{4} + \frac{v^2}{4} + \frac{v^i v_j}{2r} \right) - \frac{q^2}{r^4}$$

(4)

$$\frac{8\pi G}{c^4} \rho = \frac{\nu'}{r} e^{-\lambda} + \frac{1-e^{-\lambda}}{2r^2} - \frac{2q^2}{r^4}$$

(5)

Where prime ('') denotes the differentiation with respect to $r$ and $q$ the charge inside the radius $r$. Substituting (3) from (4) we get

$$e^{-\lambda} \left( \frac{2}{2} + \frac{\nu^2}{2r^2} - \frac{\nu'}{2r} \right) - e^{-\lambda} \lambda' \left( \frac{\nu'}{2} + \frac{1}{2r} + \frac{1}{r^2} - \Delta - 2E^2 \right) = 0$$

(6)

Where $\Delta = \frac{8\pi G}{c^4} (p_\perp - p_r)$ defined as anisotropy factor and $E = \frac{q}{r^2}$ is the electric field intensity

With the substitutions $x = Cr^2$ and $y = e^{-\lambda}$ eqn (6) reduces to

$$(1 + x^2) \frac{dy}{dx} + (2x + \frac{\nu'}{x} + x^2 \nu^2 - \frac{1}{x})y + \frac{\Delta}{x} - \frac{2E^2}{C} = 0$$

(7)

Where $\bar{v} = \frac{dy}{dx}$ and $\bar{v} + \frac{\nu'}{x} = \frac{d^2\nu}{dx^2}$

Our task is to explore the solutions of eqn (7) and obtain a physically meaningful matter distribution.

**A New Class of Solution**

To solve the above equation (7), we assume Heintzman[1] type metric potential

$$e^\nu = B(1 + x^3)$$

and we also consider $\Delta$ and $E$ of the following form:

$$\Delta = \frac{c_{max}}{1+x^3}$$

$$\frac{2B^2}{c} = \frac{2Cg^2}{x^2} = \frac{Kx}{1+x}$$

(8)
Where $k, \alpha, C, B$ are non-zero positive constants. The anisotropy and electric intensity are so assumed that the model is physically significant and well behaved i.e. $A$ and $E$ remains regular and positive throughout the sphere. In addition, both vanish at the center of gravity and increase towards the boundary.

Substituting (8) in (7) we get,

$$\frac{dy}{dx} + \frac{2x^2-x-1}{x(1+x)(1+4x)} y + \frac{1+x-x^2(a+k)}{x(1+4x)} = 0$$

(9)

which yields the following solution,

$$y = e^{-\lambda} = \frac{12x(5x^2+5a-6)+2x^2(a+k)}{12(1+x)} + \frac{Ax}{(1+x)\sqrt{1+4x}}$$

(10)

where $A$ is an arbitrary constant.

Now the expressions for density and pressures are given by

$$\rho = \frac{54+18x-a(15+15x+6x^2)-K(15+12x+12x^2)}{12(1+x)^2} - \frac{(3+9x)}{(1+x)^2(1+4x)^{1/2}} A$$

(11)

$$p_r = \frac{54-54x+a(5+37x+14x^2)+K(5+33x+20x^2)}{12(1+x)^2} + \frac{(1+x)^2(1+4x)^{1/2}}{A(1+7x)} A$$

(12)

$$p_\perp = \frac{54-54x+a(5+49x+26x^2)+K(5+33x+20x^2)}{12(1+x)^2} + \frac{(1+x)^2(1+4x)^{1/2}}{A(1+7x)} A$$

(13)

**PROPERTIES OF THE NEW SOLUTIONS**

The central values of the pressures and density is given by

$$\left[ \frac{1}{C^2} \rho \right]_{r=0} = \frac{1}{54-15K-15a-36A} > 0 \quad \text{for} \quad A < \frac{3}{2} - \frac{5(K+a)}{12}$$

(14)

$$\left[ \frac{1}{C^2} p_r \right]_{r=0} = \frac{54+5K+5a+12A}{12} > 0 \quad \text{or} \quad A > - \frac{9}{2} \frac{(K+a)}{12}$$

(15)

To satisfy the Zeldovich’s condition $[p/\rho c^2]_{r=0} \leq 1$, we have

$$\frac{54+5K+5a+12A}{54-15K-15a-36A} \leq 1 \quad \text{or} \quad A \leq - \frac{5(K+a)}{12}$$

(16)

Differentiating (14), (15) and (16) w.r.t. $x$, we get

$$\frac{1}{C^2} \frac{d\rho}{dx} = \frac{1}{12(1+x)^3} \left[ -90-18x+a(15+3x)+K(9+21x) \right] + \frac{15+69x+90x^2}{(1+4x)^2} A$$

(17)

$$\frac{1}{C^2} \frac{dp_r}{dx} = \frac{1}{12} \left[ -162+54x+a(27-9x)+K(33-3x) \right] + \frac{3-32x-42x^2}{(1+4x)^2} A$$

(18)

$$\frac{1}{C^2} \frac{dp_\perp}{dx} = \frac{1}{12} \left[ -162+54x+a(39+3x)+K(33-3x) \right] + \frac{3-32x-42x^2}{(1+4x)^2} A$$

(19)

Also the second-order differentiation for pressures and density are negative as

$$\left( \frac{1}{C^2} \frac{d^2\rho}{dx^2} \right)_{r=0} < 0, \quad \left( \frac{1}{C^2} \frac{d^2p_r}{dx^2} \right)_{r=0} < 0, \quad \left( \frac{1}{C^2} \frac{d^2p_\perp}{dx^2} \right)_{r=0} < 0$$

(20)

Hence both the pressures and density are maximum at the center and decreasing outward.

In the light of (17), (18), and (19), the radial speed of sound and the transversal speed of sound can be determined by

$$v_r^2 = \frac{dp_r}{dx}/C^2 \frac{d\rho}{dx} \quad \text{and} \quad v_\perp^2 = \frac{dp_\perp}{dx}/C^2 \frac{d\rho}{dx}$$

(21)

The ratio trace of energy stress tensor to energy density is defined as $Q = (p_r + 2p_\perp)/\rho$

(22)

Now the expression for gravitational red-shift and adiabatic index $\gamma$ are given as

$$z = e^{-\gamma} = 1 - \frac{(1+x)^2}{\sqrt{B}} - 1 \quad \text{and} \quad \gamma = \frac{dp}{d\rho}/\rho$$

(23)

Since the central value of gravitational red-shift has to be non-zero, positive finite, we have $0 < \sqrt{B} < 1$.

Differentiating (23) w.r.t. $x$ we get,

$$\left( \frac{dz}{dx} \right)_{x=0} = -\frac{3}{2\sqrt{B}} < 0$$

(24)
Similarly the derivatives of electric field and anisotropy at the center are given as
\[
\left. \frac{dE^r}{dr} \right|_{r=0} = 0 \quad \text{and} \quad \left. \frac{d\alpha}{dr} \right|_{r=0} = C\alpha > 0
\]  
(25)

Eqn (25) signifies that electric field and anisotropy are minimum (i.e. zero) at the center and monotonically increasing outward.

**BOUNDARY CONDITIONS**

The interior solution so obtained are matched with the exterior solution of Reissner-Nordström solution given by
\[
d^2s^2 = \left(1 - \frac{2GM}{r^2} + \frac{\alpha^2}{r^2} \right) c^2 dt^2 - \left(1 - \frac{2GM}{r^2} + \frac{\alpha^2}{r^2} \right)^{-1} dr^2 - r^2 \left( \text{d}\theta^2 + \sin^2 \theta \text{d}\phi^2 \right)
\]  
(26)

where \( M \) is the mass of the fluid ball as determined by the external observer and \( r_b \) is the radial coordinate of the exterior region. Since the eq. (26) is considered as the exterior solution, we shall arrive at the following conclusions by matching with (1):
\[
e^\nu = 1 - \frac{2GM}{r^2} + \frac{\alpha^2}{r^2}
\]  
(27)
\[
q (at \ r = r_b) = e
\]  
(28)
\[
e^{-\lambda} = 1 - \frac{2GM}{r^2} + \frac{\alpha^2}{r^2}
\]  
(29)
\[
p_e (r = r_b) = 0
\]  
(30)

Using (30), the value of \( A \) is obtained as
\[
A = \frac{1 + \lambda X}{1 + \frac{1}{7}X}
\]
(31)

Equations (29) and (10) yield
\[
\left(1 - \frac{2GM}{c^2 r_b} + \frac{\alpha^2}{r_b^2} \right)^{\frac{1+X}{1+\frac{1}{7}X}} = \frac{GM}{K} \quad (5+43X+20X^2)
\]  
(32)

From eqns (27), (8) and (32) we get
\[
B = \frac{1+X}{1+\frac{1}{7}X} \left(1 + X \right)^{-3}
\]  
(33)

In the view of (32) and (8) we arrive at the expression of mass as
\[
M = \frac{c^2 r_b}{2G} \left(6X-6X^2+2X^2(1+4X) \right)^{\frac{1}{1+X}(1+\frac{1}{7}X)}
\]  
(34)

Where \( X = C r_b^2 \)

Radius \( r_b \) can be determined from surface density \( \rho_b \) in eqn(11) as
\[
r_b^2 = \frac{X c^4}{8\pi G \rho_b} \left[ \frac{54 + 18X - \alpha (5 + 15X + 9X^2) - k(15 + 21X + 12X^2)}{12(1+X)^2} \right] - \frac{\left(3+9X\right)}{(1+\frac{1}{7}X)^2} A
\]  
(35)

Finally the equations for red shift and central red-shift are given respectively as
\[
z = B^{\frac{1}{2}} (1 + x) \quad \frac{x}{2} - 1 \quad \text{and} \quad z_0 = B^{\frac{1}{2}} - 1
\]  
(36)

**Table 1: The effect of anisotropy and charge on the maximum mass for different values of \( X \).**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>K</td>
<td>( \alpha )</td>
<td>( R_{b,} ) (km) ( \frac{M}{M_\odot} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>For ( \rho_b = 2 \times 10^{14} \text{ g/cc} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>5.5</td>
<td>15.15</td>
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<tr>
<td>3</td>
<td>3.5</td>
<td>15.10</td>
<td>1.91</td>
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<tr>
<td>0.2</td>
<td>0.8</td>
<td>2.8</td>
<td>16.9</td>
</tr>
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<td>2</td>
<td>16.78</td>
<td>3.11</td>
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<tr>
<td>4</td>
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<td>16.52</td>
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<td>3.6</td>
<td>0</td>
<td>16.64</td>
<td>4.25</td>
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Table 2: March of density, pressures, pressure to density ratios, square of sound speeds, $z$ and $\gamma$ for $X=0.1$, $a=2.4$ and $K=1$.

<table>
<thead>
<tr>
<th>$r/r_b$</th>
<th>$\rho r_b^2$</th>
<th>$p_r r_b^2$</th>
<th>$p_{\perp} r_b^2$</th>
<th>$p_r/c^2 \rho$</th>
<th>$p_{\perp}/c^2 \rho$</th>
<th>$Q$</th>
<th>$dp_r/c^2 dp$</th>
<th>$dp_{\perp}/c^2 dp$</th>
<th>$z$</th>
<th>$\gamma$</th>
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<td>0</td>
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<td>0.1316</td>
<td>0.1316</td>
<td>0.0937</td>
<td>0.0937</td>
<td>0.2810</td>
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<td>0.4375</td>
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<tr>
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<td>0.3</td>
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<td>0.0712</td>
<td>0.0759</td>
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<td>0.6</td>
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<td>0.0718</td>
<td>0.0802</td>
<td>0.0610</td>
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<td>0.2588</td>
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<td>0.7</td>
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<td>0.3380</td>
<td>5.22</td>
</tr>
<tr>
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</table>
Fig 1: March of density, pressures, pressure to density ratios, square of sound speeds, $E$, $\Delta$, $\gamma$, $Z$ etc from center to boundary for $X=0.1$, $\alpha=2.4$ and $K=1$ are shown below.

Table 3: The variation of maximum mass and radius of the fluid ball for different values of Schwarzschild parameter “$u$” for $\alpha=2.4$ and $K=1$.

<table>
<thead>
<tr>
<th>$u$</th>
<th>$\rho B^2$</th>
<th>$R^+_{(km)} \frac{M}{M_0}$ For $\rho_n = 2 \times 10^{14} \text{g/cc}$</th>
<th>$R^- (km) \frac{M}{M_0}$ For $\rho_n = 2.7 \times 10^{14} \text{g/cc}$</th>
<th>$R^- (km) \frac{M}{M_0}$ For $\rho_n = 4.6888 \times 10^{14} \text{g/cc}$</th>
<th>$Z_b$</th>
<th>$E_b$</th>
</tr>
</thead>
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<tr>
<td>0.03</td>
<td>1.637</td>
<td>6.923 \ 0.142 \</td>
<td>5.958 \ 0.122 \</td>
<td>4.521 \ 0.093 \</td>
<td>0.032</td>
<td>0.073</td>
</tr>
<tr>
<td>0.06</td>
<td>1.483</td>
<td>9.572 \ 0.385 \</td>
<td>8.239 \ 0.332 \</td>
<td>6.252 \ 0.252 \</td>
<td>0.065</td>
<td>0.106</td>
</tr>
<tr>
<td>0.09</td>
<td>1.328</td>
<td>11.567 \ 0.700 \</td>
<td>9.955 \ 0.603 \</td>
<td>7.554 \ 0.457 \</td>
<td>0.103</td>
<td>0.134</td>
</tr>
<tr>
<td>0.12</td>
<td>1.174</td>
<td>13.170 \ 1.070 \</td>
<td>11.335 \ 0.921 \</td>
<td>8.601 \ 0.699 \</td>
<td>0.147</td>
<td>0.162</td>
</tr>
<tr>
<td>0.15</td>
<td>1.031</td>
<td>14.408 \ 1.456 \</td>
<td>12.400 \ 1.253 \</td>
<td>9.410 \ 0.951 \</td>
<td>0.193</td>
<td>0.187</td>
</tr>
<tr>
<td>0.18</td>
<td>0.887</td>
<td>15.436 \ 1.877 \</td>
<td>13.285 \ 1.615 \</td>
<td>10.081 \ 1.226 \</td>
<td>0.246</td>
<td>0.213</td>
</tr>
</tbody>
</table>

DISCUSSIONS AND CONCLUSIONS

From figure 1 and table 2 it is observed that the various physical parameters $\left(p_r, p_\perp, \rho, \frac{dp_r}{c^2dp_\perp}, \frac{dp_r}{c^2dp_\perp} \frac{d\rho}{c^2d\rho}, Z, Q \right)$ are positive at the center and within the limit of realistic equation of state and monotonically decreasing towards the boundary. However, the anisotropy factor, electric field and adiabatic index are minimum at the center and increase outward. Thus, the solution is well behaved for all values of $X$ lying in the range $0 < X \leq 0.3$, $\alpha$ lying in the range $0 \leq \alpha \leq 5.5$, $K$ lying in the range $0 < K \leq 8.6$ and Schwarzschild compactness parameter “$u$” lying in the range $0 < u \leq 0.379$. Since our solution is well behaved for a wide ranges of different parameters, one can model many different types of ultra-cold compact stars like quark stars and neutron stars.

From table 1, it is observed that increase in charge parameter results in increase in maximum mass but increase in anisotropy results in decrease in maximum mass. With increase in charge the extra coulombic pressure helps in supporting more mass while increase in anisotropy diverts more pressure away from radial direction thereby decreasing the mass.

In table 3, we present some models of super dense quark star and neutron stars corresponding to $X=0.1$, $\alpha=2.4$ and $K=1$ for which $u_{\text{max}}=0.18$. By assuming surface density $\rho_B = 4.6888 \times 10^{14} \text{g cm}^{-3}$ the mass and radius are...
1.226 \( M_{\odot}\), 10.081 km respectively. For \( \rho_b = 2 \times 10^{14} \text{ g cm}^{-3} \) the mass and radius are 1.615 \( M_{\odot}\), 13.285 km respectively and for \( \rho_b = 2 \times 10^{14} \text{ g cm}^{-3} \) the mass and radius are 1.877 \( M_{\odot}\), 15.436 km respectively.

The well behaved class of relativistic stellar models obtained in this work might have astrophysical significance in the study of more realistic internal structures of compact stars.

REFERENCES

2. Usov VV; Electric fields at the quark surface of strange stars in the color-flavor locked phase. Phys. Rev, 2004; D70, 067301.

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