Calibration of Pipeline Friction Coefficient Based on Random Optimization
Lintao Guo
School of mathematics and statistics, Northeast Petroleum University, Daqing 163318, China.

*Corresponding Author:
Lintao Guo
Email: guolintao@126.com

Abstract: The changed friction resistance coefficient of the pipeline after the oilfield water injection pipe network system is used for a long term cannot reflect the actual conditions of the pipeline accurately. As the main parameter in the water injection system, the friction resistance coefficient of the pipeline concerns to the accuracy of the simulating calculation of water injection system and the efficiency of operation optimization.

Keywords: Random optimization; Expectation value model; Friction coefficient.

INTRODUCTION
In terms of different purposes and requirements, as for random variables during the random programming, methods are not identical completely. The most common method is as follows: a mathematic expectation of corresponding function of a random variable is taken as target function, so as to convert the random programming into a confirmed mathematic programming problem. The probability expectation of the target function reaches the optimal model (usually expectation value model) under the restriction of the expectation value [1-3].

Overview of random programming
Actually, there are a large number of unconfirmed problems, of which a random problem is available. The quantity which describes the random phenomenon is named as random variable, and mathematic programming containing random parameters is referred to as random programming[4-5].

Generation of random numbers
$x$ is assumed as random variable, probability distribution $F(\xi)$ and inverse function $F^{-1}(\xi)$ are defined on $[0,1]$. If $u$ is assumed as random variable distributed uniformly on $[0,1]$, then

$$pr\left\{F^{-1}(u) \leq y\right\} = pr\left\{u \leq F(y)\right\} = F(y)$$

$$x = F^{-1}(u)$$

(1)

(2)

Where, the probability distribution function must be $F(\xi)$. In case of generation of random number subject to $F(\xi)$, it only requires a random number $u$ distributed uniformly on $[0,1]$, which is educed by $F^{-1}(u)$.

Random simulation
The basic thought of random simulation aims to establish a probability model or random process at first, so that its parameters equal to solutions of the problems, then the statistical characteristics of the parameters calculated by observation of the model or process or sampling test, and finally the approximate values of solutions are given. The application range of the random simulation is excessively wide, and its two applications are described as follows.

Mathematic expectation of random variable function
If the distribution function $\xi$ is $P(\xi)$ and $\eta = g(\xi)$, the mathematic expectation $E(\eta)$ of $\eta$ is educed.

$$E(\eta) = \int_{\Omega} g(\xi) dP(\xi)$$

The value of $E(\eta)$ is estimated as
\[ \theta = \frac{1}{N} \sum_{i=1}^{N} g(\xi_i) \]  
\hspace{1cm} (4)

Where, \( \xi_1, \xi_2, \ldots, \xi_N \) shows random variables generated by \( \Phi(\xi) \), \( N \) shows the number of samples.

**Probability of estimated event**

It is known that the distribution function \( \xi \) is \( \Phi(\xi) \), \( g_j(\xi) \) is real-valued function, \( j = 1, 2, \ldots, k \), the following is educed

\[ \theta = \Pr \left\{ g_j(\xi) \leq 0, j = 1, 2, \ldots, k \right\} \]  
\hspace{1cm} (5)

\( N \) random variables \( \xi \) generate from \( \Phi(\xi) \), where \( i = 1, 2, \ldots, N \), of which it is assumed that \( N' \) random variables \( \xi \) meet:

\[ g_j(\xi) \leq 0, \hspace{0.5cm} j = 1, 2, \ldots, k \]  
\hspace{1cm} (6)

Where, \( i = 1, 2, \ldots, N \). The value of \( \theta \) is estimated by the formula \( \theta = \frac{N'}{N} \) according to the law of great numbers.

**II. Expectation value model calibrated by pipeline friction coefficient**

**Establishment of expectation value model**

It is assumed that the pipeline network is equipped with \( m \) pipelines and \( n \) nodes, \( L \) shows number of operating conditions, \( C_j \) shows the friction coefficient of the \( j \)-th pipeline, \( H_j^l \) shows the pressure on the \( i \)-th node of the \( l \)-th operating conditions, \( C = (C_1, C_2, \ldots, C_m)^T \) shows the friction coefficient vector, \( H_l = (H_1^l, H_2^l, \ldots, H_n^l)^T \) shows the pressure vector of the \( l \)-th group of operating conditions and \( H_{0} = (H_{01}, H_{02}, \ldots, H_{0n})^T \) shows the actually-measured pressure vector of the \( l \)-th group of operating conditions [6-7].

During actual production, there can be some unknown node pressure conditions, it is assumed that the actually-measured pressure of \( t \) nodes is unknown after numbering nodes, the pressure of each unknown node is subject to the normal distribution \( \mathcal{N}(\mu, \sigma^2) \), \( \mu \) takes the estimated pressure of node and \( \sigma \) depends on the specific conditions. The unknown pressure value \( H_{0j} \) of the later \( t \) nodes is assumed as the random variable. The minimum expectation value of the target function is educed under the restriction on expectation value.

When some node pressures are not known, the calibrated expectation value model of pipeline friction coefficient is as follows

\[
\begin{align*}
\min_{C} E[f(C, \xi)] = & M \sum_{l=1}^{L} \sum_{j=1}^{m} (H_{l}^j(C) - H_{0j}^l)^2 \quad + \quad E \left[ \sum_{l=1}^{L} \sum_{j=n-t+1}^{n} (H_{l}^j(C) - H_{0j}^l)^2 \right] \\
\text{s.t.} \quad & C_{k}^{\min} \leq C_k \leq C_{k}^{\max}, \quad k = 1, 2, \ldots, m \\
& E(\{H_{l}^j(C) - H_{0j}^l\}) \leq \delta, \quad l = 1, 2, L; \quad j = n-t+1, n-t+2, \ldots, n
\end{align*}
\]  
\hspace{1cm} (7)

Where, \( C \) shows \( m \) dimensional decision-making vector, \( \xi = ((H_{01}, H_{11}, H_{21}, \ldots, H_{0n}, H_{1n}, H_{2n}, \ldots, H_{lm}, H_{1m}, H_{2m}, \ldots, H_{nm})) \) shows \( t \cdot L \) dimensional random vector, \( M \) shows weighing, generally taking 100 according to the numerical test.

2. Solution method for expectation value model

When \( C \) is a given value, the distribution function of \( \xi \) is assumed as \( \Phi(\xi) \) to educe the target function value \( E(f(C, \xi)) \).

\[ E(f(C, \xi)) = \int_{\xi} f(C, \xi) d\Phi(\xi) \]  
\hspace{1cm} (8)

The value of \( E(f(C, \xi)) \) is estimated by the following formula

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\[ E(f(C, \xi)) = \frac{1}{N} \sum_{i=1}^{N} f(C, \xi_i) g(\xi_i) \]  

(9)

Where, \( \xi_1, \xi_2, \ldots, \xi_N \) show random variables generated by \( \Phi(\xi) \). \( N \) shows number of samples.

Specific process of solving the expectation value model[8-9]:

1. Choose coding strategies;
2. Define the adaptive value function \( f(C) \) according to the target function;
3. Confirming the genetic strategies;
4. Randomly initialize and generate the initial group;
5. Calculate individual adaptive value \( f(C) \) in the group and renew the history optimal individual;
6. If the iterative frequency reaches the specified iteration cycle, the history optimal individual is used to replace the current one;
7. In accordance with the genetic strategies, selection and selection, crossover and mutation operators are applied to acting on the colony so as to form the next generation of colonies;
8. If the iterative frequency reaches the specified period, the current optimal individuals are searched axially;

The program is ended if satisfactory, otherwise return to step 5.

**Solution example**

Some ideal water injection pipeline network of the example is composed of two water injection stations, 16 nodes, 24 pipelines and 9 loops, the nodes with 2, 15 number are the positions of the pump station. Simulating three working conditions of the pipeline network: the pump station is totally opened and the pump station is closed successively. The simplified drawing of the water injection pipeline network is as shown in figure 1, and the base data is as shown in table 1. Network parameters is the same as the expected value model of pipe network parameters.

![Fig.1 Graph of simple pipe network](image)

If the pressures of node 3, 7 and 9 are not known yet, a conventional method is to educe the optimal solution and all opportunities are taken to restrict the programming model for solution, with calibration result of friction coefficient is shown in Table 1.

In terms of the calibration result of the friction coefficient, it is preferable to any conventional method. The average value between calibration value and actual value of the expectation value model friction coefficient is 3.75 and that of the conventional method totals 6.53.
Table 1 Correction results of friction factors

<table>
<thead>
<tr>
<th>Pipeline number</th>
<th>The conventional method to calculate the friction coefficient</th>
<th>Actual friction resistance coefficient</th>
<th>Friction calibrated by expectation value model</th>
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</table>

CONCLUSION

Aiming at some unknown pressures of flooding pipe network nodes, they are assumed as random parameters to establish random optimization model - expectation value model of friction coefficient calibration; during the establishment of the random optimization model, how to further solve the distribution obeyed by the random parameters and parameters in the distribution requires more researches.

REFERENCES