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How Are Welfare Levels Affected by Changing Transaction Efficiencies? -A Theoretical Model for Dual Opening up and Unbalanced Development

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Abstract: In this theoretical study, we revisit the old Ricardian model of comparative advantage. Following an inframarginal methodology, we build an extended theoretical model based on the concepts of comparative advantage and transaction efficiency to explain development and inequality in developing economies. According to our model, an increase in domestic transaction efficiency reduces inequality within a developing economy while an increase in international transaction efficiency enhances the overall welfare level in a developing economy. The results of our model may have important policy implications for developing economies in their policy-making.
Keywords: transaction efficiency; comparative advantage; inframarginal analysis.

INTRODUCTION

The method of inframarginal analysis, which can be viewed as a combination of marginal and total cost-benefit analysis, has so far been used by quite a few works to study the issue of division of labor. For example, Cheng, Sachs, and Yang [1], using the inframarginal methodology, incorporate technological comparative advantage and transaction costs into the Heckscher-Olin (HO) model and refine the HO theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and factor equalization theorem, where the refined core theorems can be further used to justify empirical evidence that may be at odds with the traditional core theorems. Cheng, Sachs, and Yang [2], by applying the method of inframarginal analysis to the Ricardian model, show that in a two-country two-good Ricardian model there exists a unique general equilibrium given a certain level of transaction efficiency and that the comparative statics of the equilibrium involve discontinuous jumps, that is, as transaction efficiency increases, the general equilibrium structure jumps from autarky to partial division of labor and then to complete division of labor. Zhang and Shi [3], while pointing out a dual structure of division of labor and trade that is missed by the analysis of Cheng, Sachs, and Yang [2]. investigate an interesting way of using a general equilibrium model under the framework of inframarginal analysis to describe a dual structure with underemployment in a transitional period of economic development.

Other recent theoretical works involving inframarginal analysis of division of labor include Yang [4], Wen [5], Yang and Zhang [6], Yao [7,8], Sun [9], Sun, Yang, and Yao [11,10], to name but a few. In the study of this paper, using the method of inframarginal analysis, we aim to fill a lacuna in existing theoretical literature by proposing a coherent framework to investigate underlying forces shaping development and inequality in developing economies. We build a theoretical model that is diametrically different from the neoclassical growth framework to explain development and inequality in developing economies. In this model, we deliberately avoid modeling technological progress and capital accumulation in order to find alternative mechanisms through which a developing economy achieves development. We revisit the old Ricardian model of comparative advantage. Following an inframarginal analysis of the Ricardian model by Cheng, Sachs, and Yang [2], we build an extended theoretical model based on the concepts of comparative advantage and transaction efficiency to explain development and inequality in developing economies.

The rest of this paper is organized as follows. In Section 2, we provide a brief review of Cheng, Sachs, and Yang [2]'s inframarginal analysis of the Ricardian model, focusing on the model's finding of the welfare-changing effect of transaction efficiency. In Section 3, we build our extended theoretical model, incorporating features of regional development and inequality into the general inframarginal analysis framework. Section 4 concludes.

An inframarginal analysis of the Ricardian model: a brief recapitulation

In this section we recapitulate Cheng, Sachs, and Yang [2]'s inframarginal analysis of the Ricardian Model. The methodology of inframarginal analysis will form the basis of our theoretical model in the next section.

There are two countries in the world, Country Home (H) and Country Foreign (F). Each country has only one factor of production, labor, and each country is able to produce two goods, x and y. Each individual in both countries is endowed with one unit of labor, and the total number of individuals is L in Home and L^* in Foreign. Individuals within a country are assumed to be identical. The utility function for the representative consumer-producer in Home is assumed to take the Cobb-Douglas form

$$U = (x + k\hat{x})^{\alpha} (y + k\hat{y})^{1-\alpha} \tag{1}$$

where x and y are quantities of good x and good y that are produced *and* consumed by the representative Home consumer-producer, while $\hat{x}$ and $\hat{y}$ are quantities of good x and good y that are produced in Foreign but consumed by the representative Home consumer (i.e. imported goods the Home consumer consumes). k is the transaction efficiency coefficient, $0 < k < 1$. The transaction cost is assumed to take the iceberg form: for each unit of a good bought, the buyer only receives k units of the good (i.e., when the buyer pays one dollar, he gets only k dollars' worth of the good, $0 < k < 1$); the rest is lost in transit. The transaction cost may result from different sources: costs of storage, costs of transportation, and costs of finding a transaction partner, to name but a few. Analogous to equation (1), the utility function of the representative consumer in Foreign is

$$U^* = (x^* + k\hat{x}^*)^{\alpha} (y^* + k\hat{y}^*)^{1-\alpha} \tag{2}$$

where x^* and y^* are quantities of good x and good y that are produced and consumed by the representative Foreign consumer-producer, while $\hat{x}^*$ and $\hat{y}^*$ are quantities of good x and good y that are produced in Home but consumed by the Foreign consumer (i.e. imports the Foreign consumer consumes).

The unit labor requirements for good x and good y are a_x and a_y respectively in Home and are a_x^* and a_y^* respectively in Foreign. Therefore the production function for each consumer-producer in Home is

$$x + \breve{x} = l_x / a_x,\ y + \breve{y} = l_y / a_y,\ l_x + l_y = 1 \tag{3}$$

where $\breve{x}$ and $\breve{y}$ are quantities of good x and good y produced by the representative Home consumer-producer but exported to Foreign. l_x and l_y are the fraction of labor of the consumer-producer engaged in the production of good x and good y respectively. Analogous to the equations in (3), the production function for the representative Foreign consumer-producer is

$$x^* + \breve{x}^* = l_x^* / a_x^*,\ y^* + \breve{y}^* = l_y^* / a_y^*,\ l_x^* + l_y^* = 1 \tag{4}$$

For simplicity we arbitrarily assume that Home has a comparative advantage in producing good x, that is: $a_x / a_y < a_x^* / a_y^*$. Intuitively, with a sufficiently high k (close to unity), trade is possible and desirable between the two countries. Instead, if k is sufficiently low, the two countries may find themselves better off remaining in autarky. Generally, there are three possible modes of division of labor between the two countries: (i) Each country is completely specialized in the production of the good where it has a comparative advantage, i.e. Home only produces good x and Foreign only produces good y; (ii) One country is completely specialized in the production of the good where it has a comparative advantage, while the other country produces both goods; (iii) Each country remains in autarky, producing both goods.

The general equilibrium is found in two steps. First, we find the corner equilibrium for each mode. Then we identify the range of the transaction efficiency, k , within which each corner equilibrium is the general equilibrium.

Corner equilibrium in Mode (1)

In this mode of division of labor, Home will exclusively produce and export good x, and Foreign will exclusively produce and export good y. Therefore, for Home $x, \grave{x}, \acute{y} > 0$, and $\acute{x}, y, \grave{y} = 0$; for Foreign $y^*, \breve{y}^*, \hat{x}^* > 0$, and $\hat{y}^*, x^*, \breve{x}^* = 0$. The decision-making of the representative consumer in Home is described by

$$MaxU = x^{\alpha}(k\hat{y})^{1-\alpha}, \tag{5}$$

subject to $x + \grave{x} = 1/a_x$, $\acute{y} = p\grave{x}$, where $p \equiv p_x / p_y$

The maximization of U in (5) leads to

$$\breve{x} = \frac{1-\alpha}{a_x}, x = \frac{\alpha}{a_x}, \hat{y} = p \cdot \frac{1-\alpha}{a_x} \tag{6}$$

Analogous to (5), the decision-making of the representative foreign consumer is described by

$$MaxU^* = (k\hat{x}^*)^{\alpha} y^{*1-\alpha}, \tag{7}$$

subject to $y^* + \breve{y}^* = 1/a_y^*$, $\breve{y}^* = p\hat{x}^*$

The maximization problem in (7) leads to

$$\breve{y}^* = \frac{\alpha}{a_y^*}, \quad y^* = \frac{1-\alpha}{a_y^*}, \quad \hat{x}^* = \frac{1}{p} \cdot \frac{\alpha}{a_y^*} \tag{8}$$

In equilibrium, $\breve{x}L = \hat{x}^* L^*$ (and also $\hat{y}L = \breve{y}^* L^*$), from which we can solve for the equilibrium relative price p as

$$p = \frac{\alpha}{1-\alpha} \cdot \frac{L^*/a_y^*}{L/a_x} \tag{9}$$

Therefore, in equilibrium the individual utility levels in Home and Foreign are respectively

$$U = \alpha \cdot \left(\frac{1}{a_x}\right)^{\alpha} \cdot \left(\frac{kL^*}{a_y^* L}\right)^{1-\alpha}, \; U^* = (1-\alpha) \cdot \left(\frac{kL}{a_x L^*}\right)^{\alpha} \cdot \left(\frac{1}{a_y^*}\right)^{1-\alpha} \tag{10}$$

Corner equilibrium in Mode (2)

In this mode of division of labor, one country will specialize in producing the one good where it has a comparative advantage while the other country will produce both goods. This is further divided into two sub-modes: Mode (2a) and Mode (2b).

Mode (2a)

In this mode, we assume Home produces both good x and good y while Foreign completely specializes in the production of good y (where it has been assumed to have a comparative advantage). In Mode (2a), it can be seen for Home, $x, \grave{x}, y, \acute{y} > 0$, $\acute{x}, \grave{y} = 0$, while for Foreign, $y^*, \breve{y}^*, \hat{x}^* > 0$, $\hat{y}^*, x^*, \breve{x}^* = 0$. The maximization problem for the Home consumer is now

$$MaxU = x^{\alpha}(y + k\hat{y})^{1-\alpha}, \tag{11}$$

subject to $x + \grave{x} = l_x / a_x$, $y = l_y / a_y$, $l_x + l_y = 1$, $\acute{y} = p\grave{x}$

In order for Home to produce both goods, p must be such that $p = a_x /(ka_y)$. The maximization of U in (11) requires

$$\frac{-\alpha}{l_x / a_x - \breve{x}} + \frac{(1-\alpha)kp}{(1-l_x)/a_y + kp\breve{x}} = 0 \tag{12}$$

Solving for $\grave{x}$ and inserting the result back into the constraints, we obtain

$$p = a_x /(ka_y), \; \grave{x} = (l_x - \alpha)/a_x, \; x = \alpha / a_x, \tag{13}$$

$y = (1-l_x)/a_y$, $\hat{y} = (l_x - \alpha)/(k a_y)$

The utility maximization problem for the representative Foreign consumer is

$$MaxU^* = (k\hat{x}^*)^\alpha y^{*1-\alpha}, \tag{14}$$

subject to $y^* + \breve{y}^* = 1/a_y^*$, $\breve{y}^* = p\hat{x}^*$

The first-order condition in (14) requires $\frac{\alpha k}{k\hat{x}^*} = \frac{(1-\alpha)p}{1/a_y^* - p\hat{x}^*}$ which in turn implies

$$\hat{x}^* = \frac{\alpha k a_y}{a_x a_y^*},\ \breve{y}^* = \frac{\alpha}{a_y^*},\ y^* = \frac{1-\alpha}{a_y^*} \tag{15}$$

In equilibrium $\breve{x}L = \hat{x}^*L^*$, therefore we have

$$\breve{x}L = \hat{x}^*L^* \Leftrightarrow \frac{l_x-\alpha}{a_x}\cdot L = \frac{\alpha k a_y}{a_x a_y^*}\cdot L^* \Rightarrow l_x = \frac{\alpha k a_y L^*}{a_y^* L} + \alpha \tag{16}$$

As $l_x = \frac{\alpha k a_y L^*}{a_y^* L} + \alpha < 1$ if and only if $k < \frac{1-\alpha}{\alpha}\cdot\frac{L/a_y}{L^*/a_y^*}$, we have to require that $k < \frac{1-\alpha}{\alpha}\cdot\frac{L/a_y}{L^*/a_y^*}$ in order for l_x to be less than one. In equilibrium, it is straightforward to see that the individual utility levels in Home and Foreign are respectively

$$U = \left(\frac{\alpha}{a_x}\right)^\alpha \cdot \left(\frac{1-\alpha}{a_y}\right)^{1-\alpha},\ U^* = \left(\frac{\alpha k^2 a_y}{a_x a_y^*}\right)^\alpha \left(\frac{1-\alpha}{a_y^*}\right)^{1-\alpha} \tag{17}$$

Mode (2b)

In this mode of division of labor, Home only produces and exports good x, where it has a comparative advantage, while Foreign produces both good x and good y. In Mode (2b), for Home $x, \breve{x}, \hat{y} > 0$, $\hat{x}, y, \breve{y} = 0$ while for Foreign $x^*, \hat{x}^*, y^*, \breve{y}^* > 0$, $\breve{x}^*, \hat{y}^* = 0$. The utility maximization problem for the representative Home consumer is $MaxU = x^\alpha (k\hat{y})^{1-\alpha}$, subject to $x + \breve{x} = 1/a_x$ and $\hat{y} = p\breve{x}$, while that for the representative Foreign consumer is $MaxU^* = (x^* + k\hat{x}^*)^\alpha y^{*1-\alpha}$, subject to $x^* = l_x^*/a_x^*$, $y^* + \breve{y}^* = l_y^*/a_y^*$, $l_x^* + l_y^* = 1$ and $\breve{y}^* = p\hat{x}^*$. Following the same procedure as in Mode (2a), we can obtain

$$p = k a_x^*/a_y^*,\ \breve{y}^* = \frac{\alpha - l_x^*}{a_y^*},\ y^* = \frac{1-\alpha}{a_y^*},\ \hat{x}^* = \frac{\alpha - l_x^*}{k a_x^*}, \tag{18}$$

$$\breve{x} = \frac{1-\alpha}{a_x},\ x = \frac{\alpha}{a_x},\ \hat{y} = \frac{k(1-\alpha)a_x^*}{a_x a_y^*},\ l_x^* = \alpha - \frac{k(1-\alpha)a_x^* L}{a_x L^*}.$$

For $0 < l_x^* < 1$ to hold, we have to require that $k < \frac{\alpha}{1-\alpha}\cdot\frac{L^*/a_x^*}{L/a_x}$. In equilibrium, the individual utility levels in Home and Foreign are respectively

$$U = \left(\frac{\alpha}{a_x}\right)^\alpha \left(\frac{k^2(1-\alpha)a_x^*}{a_x a_y^*}\right)^{1-\alpha},\ U^* = \left(\frac{\alpha}{a_x^*}\right)^\alpha \left(\frac{1-\alpha}{a_y^*}\right)^{1-\alpha} \tag{19}$$

Corner equilibrium in Mode (3)

It is possible that both countries choose to remain in autarky. In this case, both countries obviously produce both goods. The utility maximization problem for the Home consumer is then $Max U = x^{\alpha} y^{1-\alpha}$, subject to $x = l_x / a_x$, $y = l_y / a_y$, and $l_x + l_y = 1$. It is easy to see that in equilibrium $x = \alpha / a_x$ and $y = (1-\alpha)/a_y$. Analogously, in equilibrium for Foreign, we have $x^* = \alpha / a_x^*$, $y^* = (1-\alpha)/a_y^*$. Therefore in equilibrium, the individual utility levels in Home and Foreign are:

$$U = (\alpha / a_x)^{\alpha} [(1-\alpha)/a_y]^{1-\alpha}, \ U^* = (\alpha / a_x^*)^{\alpha} [(1-\alpha)/a_y^*]^{1-\alpha} \qquad (20)$$

The general equilibrium

To ease the exposition, we define

$$k_a \equiv \left(\frac{a_x a_y}{a_x^* a_y^*}\right)^{1/2}, \ k_b \equiv \frac{1-\alpha}{\alpha} \cdot \frac{L}{L^*}, \ k_0 \equiv \left(\frac{a_x / a_y}{a_x^* / a_y^*}\right)^{1/2}, \qquad (21)$$

$$k_1 \equiv \frac{1-\alpha}{\alpha} \cdot \frac{L/a_y}{L^*/a_y^*}, \ k_2 \equiv \frac{\alpha}{1-\alpha} \cdot \frac{L^*/a_x^*}{L/a_x},$$

The general equilibrium modes can be summarized as follows.[1] If $0 < k \le k_0$, the general equilibrium structure is Mode (3), with both countries remaining in autarky. If $k_a < k_b$, and $k_0 \le k < k_1$, then the general equilibrium structure is Mode (2a), with Home producing both good x and good y while Foreign completely specializing in the production of good y. If $k_a < k_b$, and $k_1 \le k < 1$, then the general equilibrium structure is Mode (1), with the two countries engaging in complete specialization according to their respective comparative advantage. If $k_a > k_b$, and $k_0 \le k < k_2$, then the general equilibrium structure is Mode (2b), where Foreign produces both good x and good y while Home completely specializes in the production of good x. If $k_a > k_b$, and $k_2 \le k < 1$, then the general equilibrium structure is Mode (1) with the two countries engaging in complete specialization according to their respective comparative advantage.[2]

These results show that when k increases from a low value to k_0, and further to k_1 or k_2, the general equilibrium will then jump from complete autarky (Mode (3)) to incomplete division of labor (Mode (2a) or (2b)) and finally to complete specialization (Mode (1)). Whether the transitional structure is Mode (2a) or Mode (2b) depends on the relative size (as indicated by L/L^*), the relative productivity (as indicated by a_x^*/a_x, a_y^*/a_y) of the two countries, and individuals' relative preference for the two goods (as indicated by $\alpha/(1-\alpha)$). The major point of all this analysis is that the level of transaction efficiency k really *does* matter in determining the pattern of division of labor and hence the pattern of trade between two countries. As a general result, the economy develops as the transaction efficiency k increases from a sufficiently low level to a sufficiently high level. In this simple model, transaction efficiency is the final determinant for the level of development of the economy. Unlike in the neoclassical growth models, here in this model we do not need "technological progress" or capital accumulation to explain changes in the economy.

[1] We leave out the derivation procedure. Readers who are not familiar with this inframarginal analysis can refer to the next section to see a similar derivation procedure in our extended model.

[2] If $k_a = k_b$ happens to hold, then we will have $k_0 = k_1 = k_2$. This simply implies that under the condition $k_a = k_b$, the two countries will either be in complete autarky or in complete specialization, depending on the value of the actual k: if $0 < k \le k_0$, the two countries will remain in autarky; if $k_0 \le k < 1$, the two countries will engage in complete specialization according to their respective comparative advantage. Mode (2) (i.e. Mode (2a) and Mode (2b)) simply cannot be the general equilibrium structure under the condition $k_a = k_b$.

Transaction efficiency and inequality: a theoretical model

In this section, we develop a theoretical model to illustrate the impact of transaction efficiency on inequality in the welfare (utility) level. Again, there are two countries, Home (H) and Foreign (F). The general specification is the same as that of the Ricardian model in the preceding section. The difference is that in this model, Home is divided into two regions (i.e. a developed region versus a backward region), denoted H_1 and H_2 respectively. Individuals in the two regions are otherwise the same, except for their transaction efficiency with country Foreign (F).

Individuals in H_1 are assumed to have a sufficiently high transaction efficiency coefficient k such that the general equilibrium structure of division of labor between H_1 and F is Mode (1) (complete specialization with H_1 exclusively producing good x and F exclusively producing good y). Based on the results in Section 2, in order for Mode (1) between H_1 and F to be the general equilibrium structure, we have to assume here that $1 > k > k_1 \equiv \frac{1-\alpha}{\alpha}\cdot\frac{L_1/a_y}{L^*/a_y^*}$ if $\left(\frac{a_x a_y}{a_x^* a_y^*}\right)^{1/2} < \frac{1-\alpha}{\alpha}\cdot\frac{L_1}{L^*}$ and $1 > k > k_2 \equiv \frac{\alpha}{1-\alpha}\cdot\frac{L^*/a_x^*}{L_1/a_x}$ if $\left(\frac{a_x a_y}{a_x^* a_y^*}\right)^{1/2} > \frac{1-\alpha}{\alpha}\cdot\frac{L_1}{L^*}$.

In contrast, individuals in H_2 are assumed to have a low transaction efficiency coefficient k' such that no *direct* trade is possible between H_2 and F. However, (domestic) trade is possible between H_1 and H_2, and the transaction efficiency coefficient between H_1 and H_2 is assumed to be τ, where $0 < \tau < 1$. Therefore, *indirect* trade between H_2 and F is possible via H_1 if τ is not too low. It is easy to see that there are three possible modes of trade between H_1 and H_2: Mode (1^d),[3] in which H_2 exclusively produces good x and sells good x to H_1 in exchange for good y (originally produced in F) from H_1; Mode ($2a^d$), in which H_2 produces both goods and sells good x to H_1 in exchange for good y (originally produced in F) from H_1; Mode (3^d), in which H_2 is completely self-sufficient, producing both goods for itself and has no trade with H_1 at all.

Next we are going to find out the corner equilibrium for each trade mode between H_1 and H_2, given that the general equilibrium trade mode between H_1 and F has been assumed to be Mode (1).

Corner equilibrium in Mode (1^d)

In Mode (1^d), H_2 exclusively produces good x and sells good x to H_1 in exchange for good y (originally produced in F) from H_1.

The utility maximization problems for the representative individual in H_1, H_2 and F are respectively:[4]

$$MaxU_1 = (1/a_x + \tau\hat{x}_{1d} - \breve{x}_1)^{\alpha}(k\hat{y}_1 - \breve{y}_{1d})^{1-\alpha}, \tag{22}$$

subject to $\hat{y}_1 = p\breve{x}_1$, $\breve{y}_{1d} = p_d\hat{x}_{1d}$

$$MaxU_2 = (1/a_x - \breve{x}_{2d})^{\alpha}(\tau\hat{y}_{2d})^{1-\alpha}, \tag{23}$$

subject to $\hat{y}_{2d} = p_d\breve{x}_{2d}$

$$MaxU^* = (k\hat{x}^*)^{\alpha}(1/a_y^* - \breve{y}^*)^{1-\alpha}, \tag{24}$$

subject to $\breve{y}^* = p\hat{x}^*$

For the representative individual in H_1, the first-order condition requires

[3] The superscript d stands for "domestic".

[4] The subscript *d* in the following mathematical equations denotes the domestic market within country H. For example, $\hat{x}_{1d}$ denotes the quantity of good x a representative individual in H_1 buys from H_2.

$$\frac{-\alpha}{1/a_x+\tau\hat{x}_{1d}-\breve{x}_1}+\frac{(1-\alpha)kp}{kp\breve{x}_1-p_d\hat{x}_{1d}}=0 \tag{25}$$

which leads to

$$\breve{x}_1=\frac{1-\alpha}{a_x}+\left((1-\alpha)\tau+\frac{\alpha p_d}{kp}\right)\hat{x}_{1d},\ \hat{y}_1=\left[\frac{1-\alpha}{a_x}+\left((1-\alpha)\tau+\frac{\alpha p_d}{kp}\right)\hat{x}_{1d}\right]p \tag{26}$$

Similarly, for a representative individual in H_2, the first-order condition leads to

$$\breve{x}_{2d}=\frac{1-\alpha}{a_x},\ \hat{y}_{2d}=\frac{(1-\alpha)p_d}{a_x} \tag{27}$$

For the representative individual in F, the first-order condition leads to

$$\hat{x}^*=\frac{1}{p}\cdot\frac{\alpha}{a_y^*},\ \breve{y}^*=\frac{\alpha}{a_y^*} \tag{28}$$

In equilibrium, we must have $\breve{x}_1 L_1=\hat{x}^* L^*$ and $\breve{x}_{2d}L_2=\hat{x}_{1d}L_1$. These two equations combined imply

$$\hat{x}_{1d}=\frac{(1-\alpha)L_2}{a_x L_1}, \tag{29}$$

$$\breve{x}_1=\frac{1}{p}\cdot\frac{\alpha L^*}{a_y^* L_1}=\frac{1-\alpha}{a_x}+\left((1-\alpha)\tau+\frac{\alpha p_d}{kp}\right)\frac{(1-\alpha)L_2}{a_x L_1}$$

In equilibrium, it is easy to show that if an H_1 individual is willing to buy good y from F at the (relative) price p and resell it to H_2 at the (relative) price p_d, the domestic relative price p_d must be such that $p_d=\tau kp$. Inserting this back into the second equation in (29) and rearranging, we end up with

$$p=\frac{\alpha a_x L^*}{(1-\alpha)a_y^*(L_1+\tau L_2)},\ p_d=\frac{\tau k\alpha a_x L^*}{(1-\alpha)a_y^*(L_1+\tau L_2)}, \tag{30}$$

$$\breve{x}_1=\frac{(1-\alpha)(L_1+\tau L_2)}{a_x L_1},\ \hat{y}_1=\frac{\alpha L^*}{a_y^* L_1}$$

The utility levels of individuals in H_1, H_2 and F can now be calculated as

$$U_1=\alpha\left(\frac{1}{a_x}\right)^{\alpha}\left(\frac{kL^*}{a_y^*(L_1+\tau L_2)}\right)^{1-\alpha},\ U_2=\alpha\left(\frac{1}{a_x}\right)^{\alpha}\left(\frac{k\tau^2 L^*}{a_y^*(L_1+\tau L_2)}\right)^{1-\alpha}, \tag{31}$$

$$U^*=(1-\alpha)\left(\frac{k(L_1+\tau L_2)}{a_x L^*}\right)^{\alpha}\left(\frac{1}{a_y^*}\right)^{1-\alpha}$$

Corner equilibrium in Mode (2a^d)

In Mode ($2a^d$), H_2 produces both goods and sells good x to H_1 in exchange for good y (originally produced in F) from H_1. The utility maximization problems for the representative individual in H_1, H_2 and F are respectively

$$MaxU_1=(1/a_x+\tau\hat{x}_{1d}-\breve{x}_1)^{\alpha}(k\hat{y}_1-\breve{y}_{1d})^{1-\alpha}, \tag{32}$$

subject to $\hat{y}_1=p\breve{x}_1,\ \breve{y}_{1d}=p_d\hat{x}_{1d}$

$$MaxU_2=\left(\frac{l_{2x}}{a_x}-\breve{x}_{2d}\right)^{\alpha}\left(\frac{1-l_{2x}}{a_y}+\tau\hat{y}_{2d}\right)^{1-\alpha}, \tag{33}$$

subject to $\hat{y}_{2d} = p_d \breve{x}_{2d}$

$$MaxU^* = (k\hat{x}^*)^\alpha (1/a_y^* - \breve{y}^*)^{1-\alpha}, \quad (34)$$

subject to $\breve{y}^* = p\hat{x}^*$

Since H_2 produces both goods, in equilibrium we must have $p_d = \frac{a_x}{\tau a_y}$. Following the same procedure as above, we get

$$\breve{x}_1 = \frac{1-\alpha}{a_x} + \left((1-\alpha)\tau + \frac{\alpha p_d}{kp}\right)\hat{x}_{1d}, \; \hat{y}_1 = \left[\frac{1-\alpha}{a_x} + \left((1-\alpha)\tau + \frac{\alpha p_d}{kp}\right)\hat{x}_{1d}\right]p, \quad (35)$$

$$\breve{x}_{2d} = \frac{l_{2x} - \alpha}{a_x}, \; \hat{y}_{2d} = \frac{l_{2x} - \alpha}{\tau a_y}, \hat{x}^* = \frac{1}{p} \cdot \frac{\alpha}{a_y^*}, \; \breve{y}^* = \frac{\alpha}{a_y^*}$$

In equilibrium, we must have $\breve{x}_1 L_1 = \hat{x}^* L^*$ and $\breve{x}_{2d} L_2 = \hat{x}_{1d} L_1$. These two equations combined imply that

$$\hat{x}_{1d} = \frac{(l_{2x} - \alpha)L_2}{a_x L_1}, \quad (36)$$

$$\breve{x}_1 = \frac{1}{p} \cdot \frac{\alpha L^*}{a_y^* L_1} = \frac{1-\alpha}{a_x} + \left((1-\alpha)\tau + \frac{\alpha p_d}{kp}\right)\frac{(l_{2x} - \alpha)L_2}{a_x L_1}$$

Again, in equilibrium, we must have $p_d = \tau kp$, using this and inserting $p_d = \frac{a_x}{\tau a_y}$ back into the second equation in (36), we end up with

$$p = \frac{a_x}{k\tau^2 a_y}, \; l_{2x} = \frac{\alpha k \tau a_y L^*}{a_y^* L_2} - \frac{(1-\alpha)L_1}{\tau L_2} + \alpha \quad (37)$$

Since H_2 produces both goods, we must have $l_{2x} < 1$, therefore we must have

$$\tau < \frac{L_2 + \sqrt{L_2{}^2 + 4\alpha k a_y L^* L_1 / [(1-\alpha)a_y^*]}}{2\alpha k a_y L^* / [(1-\alpha)a_y^*]} \quad (38)$$

Further, $l_{2x} \geq \alpha$ must hold (since α is the equilibrium labor input in the production of good x under autarky), which implies that $\tau \geq \left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2}$ (With our earlier assumptions concerning k, it is easy to see that $\left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2} < 1$).

It is now easy to obtain the utility levels of individuals in H_1, H_2 and F as

$$U_1 = \left(\frac{\alpha}{a_x}\right)^\alpha \left(\frac{1-\alpha}{\tau^2 a_y}\right)^{1-\alpha}, \; U_2 = \left(\frac{\alpha}{a_x}\right)^\alpha \left(\frac{1-\alpha}{a_y}\right)^{1-\alpha}, \quad (39)$$

$$U^* = \left(\frac{k^2 \tau^2 \alpha a_y}{a_x a_y^*}\right)^\alpha \left(\frac{1-\alpha}{a_y^*}\right)^{1-\alpha}$$

Corner equilibrium in Mode (3^d)

In Mode (3^d), H_2 is completely self-sufficient, producing both goods for itself and has no trade with H_1 at all. Based on the relevant analysis in Section 2, it is now easy to find that in this case

$$x_1 = \frac{1-\alpha}{a_x},\ y_1 = p\cdot\frac{1-\alpha}{a_x},\ x^* = \frac{1}{p}\cdot\frac{\alpha}{a_y^*},\ y^* = \frac{\alpha}{a_y^*},\ p = \frac{\alpha}{1-\alpha}\cdot\frac{L^*/a_y^*}{L_1/a_x} \tag{40}$$

In equilibrium, the individual utility levels in H_1, H_2 and F are respectively

$$U_1 = \alpha\cdot\left(\frac{1}{a_x}\right)^{\alpha}\left(\frac{kL^*}{a_y^* L_1}\right)^{1-\alpha},\ U_2 = \left(\frac{\alpha}{a_x}\right)^{\alpha}\left(\frac{1-\alpha}{a_y}\right)^{1-\alpha}, \tag{41}$$

$$U^* = (1-\alpha)\left(\frac{kL_1}{a_x L^*}\right)^{\alpha}\left(\frac{1}{a_y^*}\right)^{1-\alpha}$$

The general equilibrium

Now we can turn to a discussion of the resulted general equilibrium structures based on the interactions of individuals in H_1, H_2 and F. First, consider the equilibrium in Mode ($2a^d$). In order for the equilibrium in Mode ($2a^d$) to be the general equilibrium structure, then at the equilibrium relative price $p = \frac{a_x}{k\tau^2 a_y}$, individuals in F must prefer complete specialization in good y to autarky. Thus, the condition $\left(\frac{k^2\tau^2\alpha a_y}{a_x a_y^*}\right)^{\alpha}\left(\frac{1-\alpha}{a_y^*}\right)^{1-\alpha} \geq \left(\frac{\alpha}{a_x^*}\right)^{\alpha}\left(\frac{1-\alpha}{a_y^*}\right)^{1-\alpha}$ must hold, which implies $\tau \geq \left(\frac{a_x a_y^*}{k^2 a_x^* a_y}\right)^{1/2}$. Comparing this result with our earlier requirement that $\tau \geq \left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2}$, under the assumptions we have made concerning the value of k, it is easy to see that: $\left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2} > \left(\frac{a_x a_y^*}{k^2 a_x^* a_y}\right)^{1/2}$. Also, at $p_d = \frac{a_x}{\tau a_y}$, individuals in H_2 must prefer Mode ($2a^d$) to complete specialization in good x *and* to autarky. That is

$$\left(\frac{\alpha}{a_x}\right)^{\alpha}\left(\frac{1-\alpha}{a_y}\right)^{1-\alpha} \geq \left(\frac{\alpha}{a_x}\right)^{\alpha}\left(\frac{\tau(1-\alpha)p_d}{a_x}\right)^{1-\alpha} \Leftrightarrow \frac{a_x}{a_y} \geq \tau p_d = \frac{a_x}{a_y},$$

which holds automatically, as well as $\left(\frac{\alpha}{a_x}\right)^{\alpha}\left(\frac{1-\alpha}{a_y}\right)^{1-\alpha} \geq \left(\frac{\alpha}{a_x}\right)^{\alpha}\left(\frac{1-\alpha}{a_y}\right)^{1-\alpha}$, which holds trivially. Still, at $p = \frac{a_x}{k\tau^2 a_y}$ and $p_d = \frac{a_x}{\tau a_y}$, individuals in H_1 must prefer Mode ($2a^d$) to Mode (3^d) concerning trade with H_2. Therefore, we should have

$$\left(\frac{\alpha}{a_x}\right)^{\alpha}\left(\frac{1-\alpha}{\tau^2 a_y}\right)^{1-\alpha} \geq \left(\frac{\alpha}{a_x}\right)^{\alpha}\left(k\cdot\frac{a_x}{k\tau^2 a_y}\cdot\frac{1-\alpha}{a_x}\right)^{1-\alpha},$$

which holds automatically.

Therefore, in order for the (corner) equilibrium in Mode (2a[d]) to be the general equilibrium structure, we have to require that

$$\tau_0 \equiv \left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2} \le \tau < \frac{L_2 + \sqrt{L_2{}^2 + 4\alpha k a_y L^* L_1 /[(1-\alpha)a_y^*]}}{2\alpha k a_y L^* /[(1-\alpha)a_y^*]} \equiv \tau_1 \tag{42}$$

Obviously, a prerequisite for the inequality in (42) to hold is

$$\left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2} < \frac{L_2 + \sqrt{L_2{}^2 + 4\alpha k a_y L^* L_1 /[(1-\alpha)a_y^*]}}{2\alpha k a_y L^* /[(1-\alpha)a_y^*]}$$

which can be shown to hold automatically.

In addition, for $\tau_1 \equiv \frac{L_2 + \sqrt{L_2{}^2 + 4\alpha k a_y L^* L_1 /[(1-\alpha)a_y^*]}}{2\alpha k a_y L^* /[(1-\alpha)a_y^*]} < 1$ to hold, it is easy to show that we have to further assume

$$k > \frac{1-\alpha}{\alpha} \cdot \frac{L/a_y}{L^*/a_y^*} \tag{43}$$

where $L \equiv L_1 + L_2$.

Now, in order for the (corner) equilibrium in Mode (1[d]) to be the general equilibrium structure, then at the corner equilibrium relative price $p = \frac{\alpha a_x L^*}{(1-\alpha)a_y^*(L_1 + \tau L_2)}$, individuals in F must prefer complete specialization in good y to autarky. Thus, we must have

$$\tau \ge \frac{\alpha a_x L^*}{k(1-\alpha)a_x^* L_2} - \frac{L_1}{L_2} \tag{44}$$

With our earlier assumptions concerning k, it is easy to see that $\frac{\alpha a_x L^*}{k(1-\alpha)a_x^* L_2} - \frac{L_1}{L_2} < 0$. Therefore the condition in (44) holds automatically.

Also, for the equilibrium in Mode (1[d]) to be the general equilibrium structure, at $p_d = \frac{\tau k \alpha a_x L^*}{(1-\alpha)a_y^*(L_1 + \tau L_2)}$, individuals in H_2 must prefer Mode (1[d]) to autarky, which requires

$$\tau \ge \frac{L_2 + \sqrt{L_2{}^2 + 4\alpha k a_y L^* L_1 /[(1-\alpha)a_y^*]}}{2\alpha k a_y L^* /[(1-\alpha)a_y^*]} \tag{45}$$

Still, at $p = \frac{\alpha a_x L^*}{(1-\alpha)a_y^*(L_1 + \tau L_2)}$ and $p_d = \frac{\tau k \alpha a_x L^*}{(1-\alpha)a_y^*(L_1 + \tau L_2)}$, individuals in H_1 must prefer Mode (1[d]) to Mode (3[d]) concerning trade with H_2. Therefore, we should have

$$\alpha\left(\frac{1}{a_x}\right)^{\alpha}\left(\frac{kL^*}{a_y^*(L_1 + \tau L_2)}\right)^{1-\alpha} \ge \left(\frac{\alpha}{a_x}\right)^{\alpha}\left(k \cdot \frac{\alpha a_x L^*}{(1-\alpha)a_y^*(L_1 + \tau L_2)} \cdot \frac{1-\alpha}{a_x}\right)^{1-\alpha},$$

which holds automatically. Therefore, in order for the equilibrium in Mode (1[d]) to be the general equilibrium structure, we have to require that

$$\tau \geq \frac{L_2 + \sqrt{L_2{}^2 + 4\alpha k a_y L^* L_1 /[(1-\alpha)a_y^*]}}{2\alpha k a_y L^* /[(1-\alpha)a_y^*]} \equiv \tau_1 \tag{46}$$

Now consider the (corner) equilibrium in Mode (3[d]). In order for this to be the general equilibrium structure, we have to make a series of comparisons. First, at the corner equilibrium relative prices $p = \frac{\alpha a_x L^*}{(1-\alpha)a_y^* L_1}$ and $p_d = \frac{\tau k \alpha a_x L^*}{(1-\alpha)a_y^* L_1}$, we compare the utility levels of H_1, H_2 and F in Mode (3[d]) against the corresponding utility levels of H_1, H_2 and F in Mode (1[d]). In order for Mode (3[d]) to be the general equilibrium structure, the following must be required:

$$\alpha \cdot \left(\frac{1}{a_x}\right)^{\alpha} \left(\frac{kL^*}{a_y^* L_1}\right)^{1-\alpha} \geq \left(\frac{\alpha}{a_x}\right)^{\alpha} \left(\frac{k\alpha L^*}{a_y^* L_1} - \frac{\tau k \alpha a_x L^*}{(1-\alpha)a_y^* L_1} \cdot \frac{(1-\alpha)L_2}{a_x L_1}\right)^{1-\alpha},$$

which holds automatically, and

$$\left(\frac{\alpha}{a_x}\right)^{\alpha} \left(\frac{1-\alpha}{a_y}\right)^{1-\alpha} \geq \left(\frac{1}{a_x} - \frac{1-\alpha}{a_x}\right)^{\alpha} \left(\tau \cdot \frac{(1-\alpha)}{a_x} \cdot \frac{\tau k \alpha a_x L^*}{(1-\alpha)a_y^* L_1}\right)^{1-\alpha}$$

which in turn implies

$$\tau \leq \left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2} \tag{47}$$

and

$$(1-\alpha)\left(\frac{kL_1}{a_x L^*}\right)^{\alpha} \left(\frac{1}{a_y^*}\right)^{1-\alpha} \geq \left(k \cdot \frac{(1-\alpha)a_y^* L_1}{\alpha a_x L^*} \cdot \frac{\alpha}{a_y^*}\right)^{\alpha} \left(\frac{1}{a_y^*} - \frac{\alpha}{a_y^*}\right)^{1-\alpha},$$

which holds automatically. Therefore, in order for Mode (3[d]) to be the general equilibrium structure, we have to require that

$$\tau \leq \left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2} \equiv \tau_0 \tag{48}$$

To sum up, we have so far reached these results. If $0 < \tau \leq \left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2} \equiv \tau_0$, the general equilibrium trade structure between H_1 and H_2 is Mode (3[d]), in which H_2 is completely self-sufficient, producing both goods for itself and has no trade with H_1 at all. If $\tau_0 \equiv \left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2} \leq \tau < \frac{L_2 + \sqrt{L_2{}^2 + 4\alpha k a_y L^* L_1 /[(1-\alpha)a_y^*]}}{2\alpha k a_y L^* /[(1-\alpha)a_y^*]} \equiv \tau_1$, the general equilibrium trade structure between H_1 and H_2 is Mode (2a[d]), in which H_2 produces both goods and sells good x to H_1 in exchange for good y (originally produced in F) from H_1. If $\tau_1 \equiv \frac{L_2 + \sqrt{L_2{}^2 + 4\alpha k a_y L^* L_1 /[(1-\alpha)a_y^*]}}{2\alpha k a_y L^* /[(1-\alpha)a_y^*]} \leq \tau < 1$, the general equilibrium trade structure between H_1 and H_2 is Mode (1[d]), in which H_2 exclusively produces good x and sells good x to H_1 in exchange for good y (originally produced in F) from H_1.

Comparative statics

Under all of our earlier assumptions concerning the value of k, we can now carry out a comparative static analysis of the individual utility levels in H_1, H_2 and F with respect to the value of τ.

When $0 < \tau \le \left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2} \equiv \tau_0$, the general equilibrium trade structure between H_1 and H_2 is Mode (3^d). The corresponding individual utility levels in H_1, H_2 and F are shown in (41). Obviously, when τ increases within the interval $(0, \tau_0]$, U_1, U_2 and U^* all remain unchanged.

When $\tau_0 \equiv \left(\frac{(1-\alpha)a_y^* L_1}{k\alpha a_y L^*}\right)^{1/2} \le \tau < \frac{L_2 + \sqrt{L_2{}^2 + 4\alpha k a_y L^* L_1 /[(1-\alpha)a_y^*]}}{2\alpha k a_y L^* /[(1-\alpha)a_y^*]} \equiv \tau_1$, the general equilibrium trade structure between H_1 and H_2 is Mode ($2a^d$). The individual utility levels in H_1, H_2 and F are shown in (39). First, at $\tau = \tau_0$, it is easily seen that there is no discontinuous jump for U_1, U_2 and U^*. Then when τ increases continuously within the interval $[\tau_0, \tau_1)$, we can see that U_1 decreases continuously, U_2 remains unchanged, and U^* increases continuously.

Finally, when $\tau_1 \equiv \frac{L_2 + \sqrt{L_2{}^2 + 4\alpha k a_y L^* L_1 /[(1-\alpha)a_y^*]}}{2\alpha k a_y L^* /[(1-\alpha)a_y^*]} \le \tau < 1$, the general equilibrium trade structure between H_1 and H_2 is Mode (1^d). The individual utility levels in H_1, H_2 and F are shown in (31). First, it is easy to see that at $\tau = \tau_1$, there is no discontinuous jump for U_1, U_2 and U^*. Then, when τ increases continuously within the interval $[\tau_1, 1)$, we can see that U_1 decreases continuously, U_2 increases continuously, and U^* increases continuously.

We can also carry out a comparative static analysis of the individual utility levels in H_1, H_2 and F with respect to the trade pattern between H_1 and F. Suppose initially the transaction efficiency coefficient k between H_1 and F is so low that in equilibrium no trade is possible between H_1 and F (that is Mode (3) between H_1 and F). We can study how the individual utility levels in H_1, H_2 and F will change if k jumps from such a low value $\underline{k}$ to a sufficiently high value $\overline{k}$ (which meets all of our earlier assumptions and with which the general equilibrium mode of trade between H_1 and F is Mode (1)).

With the initial low value $\underline{k}$, there is no trade between the two countries, and obviously there is no domestic trade between H_1 and H_2. Therefore, with any $\tau \in (0,1)$, we always have

$$U_1 = U_2 = \left(\frac{\alpha}{a_x}\right)^{\alpha}\left(\frac{1-\alpha}{a_y}\right)^{1-\alpha}, \; U^* = \left(\frac{\alpha}{a_x^*}\right)^{\alpha}\left(\frac{1-\alpha}{a_y^*}\right)^{1-\alpha} \tag{49}$$

Now, suppose the transaction efficiency coefficient k between H_1 and F jumps to a sufficiently high value $\overline{k}$ (one that meets all of our earlier assumptions). Now at $\overline{k}$, if τ happens to be such that $0 < \tau \le \left(\frac{(1-\alpha)a_y^* L_1}{\overline{k}\alpha a_y L^*}\right)^{1/2} \equiv \overline{\tau_0}$, then the individual utility levels in H_1, H_2 and F follow the equations in (41). It is easy to show that with this upward jump in k

from $\underline{k}$ to $\overline{k}$, if $0 < \tau \le \overline{\tau_0}$, then U_1 unambiguously jumps upward, U_2 remains unchanged, and U^* unambiguously jumps upward.

Instead, at the new level of the transaction efficiency coefficient $\overline{k}$, if τ happens to be such that $\overline{\tau_0} \equiv \left(\frac{(1-\alpha)a_y^* L_1}{\overline{k}\alpha a_y L^*} \right)^{1/2} \le \tau < \frac{L_2 + \sqrt{L_2^{\ 2} + 4\alpha \overline{k} a_y L^* L_1 / [(1-\alpha)a_y^*]}}{2\alpha \overline{k} a_y L^* / [(1-\alpha)a_y^*]} \equiv \overline{\tau_1}$, then the individual utility levels in H_1, H_2 and F follow the equations in (39). It is also easy to show that with the upward jump in k from $\underline{k}$ to $\overline{k}$, if $\overline{\tau_0} \le \tau < \overline{\tau_1}$, then U_1 unambiguously jumps upward, U_2 remains unchanged, and U^* unambiguously jumps upward.

Finally, at the new level of the transaction efficiency coefficient $\overline{k}$, if τ happens to be such that $\overline{\tau_1} \equiv \frac{L_2 + \sqrt{L_2^{\ 2} + 4\alpha \overline{k} a_y L^* L_1 / [(1-\alpha)a_y^*]}}{2\alpha \overline{k} a_y L^* / [(1-\alpha)a_y^*]} \le \tau < 1$, then the individual utility levels in H_1, H_2 and F follow the equations in (31). It is easy to show that with the upward jump in k from $\underline{k}$ to $\overline{k}$, if $\overline{\tau_1} \le \tau < 1$, then U_1 unambiguously jumps upward, U^* unambiguously jumps upward, and U_2 remains unchanged (if exactly $\tau = \overline{\tau_1}$) or jumps upward (if $\overline{\tau_1} < \tau < 1$).

Also, it is easy to see another related fact: if k is now sufficiently high so that the general equilibrium trade mode between H and F is Mode (1), then a marginal increase in k (one such that it does not cause the general equilibrium between H_1 and H_2 to shift from one mode to another) will leave individuals in F strictly better off, and individuals in H_1 and H_2 at least no worse off than before the increase in k .

Next, let's study the model from another perspective. Now, suppose as before, k meets all our earlier assumptions so that with this value of k, the general equilibrium trade structure between H and F is Mode (1). The domestic transaction efficiency coefficient may fall into any one of the three intervals, $(0, \tau_0]$, $[\tau_0, \tau_1)$, and $[\tau_1, 1)$. Now suppose both k and τ are fixed, but the border between H_1 and H_2 shifts so that H_1 is now larger and H_2 smaller (i.e. L_1 increases and L_2 decreases, with the total population of H fixed at $L = L_1 + L_2$). Let's now study the effects of an increase in L_1 on the individual utility levels of H_1, H_2 and F, holding both k and τ fixed. First, we have to note that a change in L_1 shifts the dividing points τ_0 and τ_1. It is not difficult to show that

$$\frac{\partial \tau_0}{\partial L_1} = \frac{1}{2} \left(\frac{(1-\alpha)a_y^*}{k\alpha a_y L^*} \right)^{1/2} L_1^{-1/2} > 0, \tag{50}$$

$$\frac{\partial \tau_1}{\partial L_1} = \frac{1}{M} \left\{ -1 + \left[(L-L_1)^2 + 2ML_1 \right]^{-1/2} (-L + L_1 + M) \right\} > 0$$

where $M \equiv 2\alpha k a_y L^* / [(1-\alpha)a_y^*]$. With our earlier assumption that $k > \frac{1-\alpha}{\alpha} \cdot \frac{L / a_y}{L^* / a_y^*}$, it is not difficult to show that $M > 2L$, which, in turn, can be easily shown to imply that $\left[(L-L_1)^2 + 2ML_1 \right]^{-1/2} (-L + L_1 + M) > 1$, hence $\frac{\partial \tau_1}{\partial L_1} > 0$.

If a marginal increase in L_1 is such an increase that it does not shift the general equilibrium trade structure between H_1 and H_2, then it is straightforward to see that this marginal increase in L_1 will make individuals in F strictly better off (if Mode (1^d) or Mode (3^d) is the general equilibrium structure between H_1 and H_2) or at least no worse off (if Mode ($2a^d$) is the general equilibrium structure between H_1 and H_2), make individuals in H_1 strictly worse off (if Mode (1^d) or Mode (3^d) is the general equilibrium structure between H_1 and H_2) or no better off (if Mode ($2a^d$) is the general equilibrium structure between H_1 and H_2), and make individuals in H_2 strictly worse off (if Mode (1^d) is the general equilibrium structure between H_1 and H_2) or no better off (if Mode ($2a^d$) or Mode (3^d) is the general equilibrium structure between H_1 and H_2).

At a sufficiently high level of k and a given level[5] of τ, suppose now there is such a discontinuous upward jump in L_1 that causes the equilibrium structure between H_1 and H_2 to shift from Mode (1^d) to Mode ($2a^d$). It is easy to see that U_1 unambiguously decreases, U_2 unambiguously decreases, and U^* unambiguously increases as a result. If instead this upward jump in L_1 has caused the equilibrium structure between H_1 and H_2 to shift from Mode ($2a^d$) to Mode (3^d), then as a result, U_1 unambiguously increases, U_2 does not change, and U^* unambiguously decreases.

In developing and transition economies, transaction efficiency has a lot to do with the infrastructure and institution, and factors underlying transaction efficiency are usually fast changing in a transition economy. Therefore, the study of (changes in) transaction efficiency in a developing and transition country is very important for revealing and explaining trade patterns of the country, as well as their effects on economic growth and development in this country.

One limit of our analysis in this section is that we have assumed exogenous transaction efficiency and exogenous comparative advantage in our model above. However, both transaction efficiency and comparative advantage can be endogenously determined within the economy. For example, if we define “full transaction efficiency” as zero transaction costs, then non-zero transaction costs will reduce the actual effects of the comparative advantage of one country (as seen from the perspective of the other country). Non-zero transaction costs would affect not only the effects of exogenous (static) comparative advantage, but also the evolution path of endogenous (dynamic) comparative advantage. Moreover, transaction efficiency can be either exogenous or endogenous too. If transaction efficiency is assumed to be given and fixed, then it is exogenous and not affected by comparative advantages, trade patterns, or other related factors. However, transaction efficiency can be endogenous as well. A country may acquire transaction efficiency just as it acquires endogenous comparative advantage by, say, knowledge accumulation. Then the patterns of comparative advantage and trade may have a lot to say about the evolution path of endogenous transaction efficiency. Therefore, in this case, effects of foreign trade on output, economic development, the welfare level, and regional disparities will depend heavily on the intricate interactions between static comparative advantage, dynamic advantage and transaction efficiency. A thorough discussion of this issue, however, is beyond the scope of this paper.

Concluding remarks

In this study, we have built a theoretical model that is diametrically different from the neoclassical growth framework to explain development and inequality in developing economies. In this model, by deliberately avoid modeling technological progress and capital accumulation, we have strived to find alternative mechanisms through which developing economies achieve their development. Following an inframarginal analysis framework of Cheng, Sachs, and Yang [2] based on the old Ricardian model, we have built an extended theoretical model based on the concepts of comparative advantage and transaction efficiency to explain development and inequality in developing economies.

Generally, according to our model, if domestic transaction efficiency, which can be viewed as a function of domestic legal, institutional and policy environment, is increased, the welfare (utility) level of H_1 households tends to decrease and the welfare level of H_2 households can be increased, thus reducing inequality between the two parts of a

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[5] For simplicity and without loss of generality, we leave out the possibility that the given τ exactly equals one of the crucial values, τ_0 and τ_1.

developing economy. In contrast, if international transaction efficiency, which can also be viewed as a function of legal, institutional and policy environment, is increased, then the welfare levels in both parts of the developing country can be increased. These and other basic results of our model may have important policy implications for developing economies in their policy-making.

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